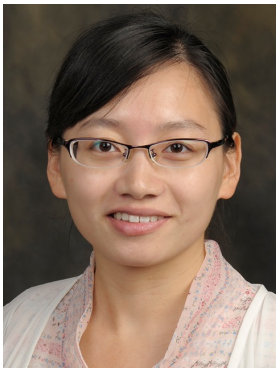
Guangzhou Discrete Mathematics Seminar

On the size of $(K_t, \mathcal{T}_k)$-co-critical graphs



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Room 416, School of Mathematics, Sun Yat-sen University

Given an integer $r \geq 1$ and graphs $G, H_1, \ldots, H_r$, we write $G \to (H_1, \ldots, H_r)$ if every r-coloring of the edges of G contains a monochromatic copy of H_i in color i for some $i \in \{1, \ldots, r\}$. A non-complete graph G is $(H_1, \ldots, H_r)$-*co-critical* if $G \not\to (H_1, \ldots, H_r)$, but $G + e \to (H_1, \ldots, H_r)$ for every edge e in $\overline{G}$. Motivated by Hanson and Toft's conjecture [Edge-colored saturated graphs, J. Graph Theory 11 (1987), 191–196], we study the minimum number of edges over all $(K_t, \mathcal{T}_k)$-co-critical graphs on n vertices, where $\mathcal{T}_k$ denotes the family of all trees on k vertices. Following Day [Saturated graphs of prescribed minimum degree, Combin. Probab. Comput. 26 (2017), 201–207], we apply graph bootstrap percolation on a not necessarily K_t-saturated graph to prove that for all $t \geq 4$ and $k \geq \max\{6, t\}$, there exists a constant $c(t, k)$ such that, for all $n \geq (t-1)(k-1)+1$, if G is a $(K_t, \mathcal{T}_k)$-co-critical graph on n vertices, then

$$e(G) \geq \left(\frac{4t-9}{2} + \frac{1}{2}\left\lceil \frac{k}{2} \right\rceil\right) n - c(t, k).$$

Furthermore, this linear bound is asymptotically best possible when $t \in \{4, 5\}$ and $k \geq 6$. The method we developed may shed some light on attacking Hanson and Toft's conjecture.

Joint work with Zi-Xia Song (University of Central Florida).